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Particle Swarm Optimization to the Retrial Machine Repair Problem with Working Breakdowns under the N Policy

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Abstract: This paper analyzes a retrial machine repair problem with working breakdowns operating under the N policy. The server is subject to working breakdowns only when there is at least one failed machine in the system. A matrix-analytic method is employed to derive the steady-state probabilities of the number of failed machines in the system which is used to evaluate several system performance measures. We construct a profit model to determine the optimum number of warm standbys S, and the joint optimal values for fast service and slow service rates, simultaneously by using Canonical particle swarm optimization (CPSO) algorithm. Moreover, we carry out the sensitivity analysis with numerical illustration based on various system parameter values.

Keywords: Canonical particle swarm optimization, matrix-analytic method, retrial machine repair problem, sensitivity analysis, working breakdowns.

1. Introduction

We investigate a retrial machine repair problem with working breakdowns operating under the N policy. Failed machines join the retrial orbit only when the server is busy or is subject to working breakdowns. The failed machines not being served at the failure instants join the retrial orbit and then try to get the service again after some random time. Working breakdowns describes that the server does not cause total breakdown, but it keeps service with a lower service rate. We assume that the server is servicing either at a fast rate (when the server is turned on and working) or at a slow rate (when the server is subject to working breakdowns). The so-called N policy is to turn the server on when N ($N \geq 1$) or more failed machines are present in the system, and turn the server off only when the system becomes empty. Retrial queueing problems play a key role in

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many applications such as telephone switching systems, telecommunication systems, redundant repairable systems, production management, inventory management, and so on.

Fayolle [13] introduced an M/M/1 retrial queue where a customer finds the server busy joining the retrial orbit and only the customer at the head of the queue can try for service after an exponential retrial time. Farahmand [12] called this discipline a retrial queue with FCFS (first-come, first-served) orbit. Falin and Templeton [11], and Artalejo and Gomez-Corral [3] provided the most comprehensive concepts for the retrial queues. Choi *et al.* [6] investigated an M/M/1 retrial queue where the retrial time has a general distribution and only the customer at the head of the queue is allowed to retry for service. They found a necessary and sufficient condition for ergodicity. Wang *et al.* [26] and [27] studied the M/M/1 retrial queue with infinite source and finite source of customers from the perspective of reliability for the first time. An unreliable M/M/1 retrial queue with infinite-capacity orbit and normal queue was examined by Sherman and Kharoufeh [23]. They provided stability conditions as well as several stochastic decomposability results. Do [7] considered an M/M/1 retrial queue with working vacations. He developed the closed-form solution in steady-state. An M/M/1 retrial queue with working vacations and negative customers was studied by Do *et al.* [8]. They presented efficient methodology to compute the stationary distribution of this retrial queue. Tao *et al.* [25] dealt with an M/M/1 retrial queue with collisions and working vacation interruption under N-policy. Using the matrix-analytic method, they developed the stationary probability distribution and some performance measures. A Markovian retrial queue with constant retrial rate and an unreliable server under the threshold recovery policy was discussed by Efrosinin and Winkler [9]. They derived mean performance characteristics and waiting time distribution as well as determined the optimal threshold recovery level. Moreover, Ke *et al.* [16] considered a repairable K-out-of-(M+W) retrial system, and Kuo *et al.* [20] also considered a retrial and repairable multi-component system with mixed warm and cold standby components. Kuo and Ke [18,19] studied the steady-state availability of a repairable system respectively. Extensive surveys and bibliographies of retrial queues are given in Yang and Templeton [29], Falin [10], and Artalejo [1, 2].

Kalidass and Kasturi [15] analyzed an infinite source queue with working breakdowns. They developed the Laplace Stieltjes transform of the distribution of waiting time in the system as well as several system performance measures. A comprehensive and excellent review on the machine repair problem (MRP) was proposed by Haque and Armstrong [14]. The particle swarm optimization (PSO) algorithm has been widely utilized by several authors such as Wang *et al.* [28], Liou [21], and so on. A multiple-vacation M/M/1 warm-standby MRP with an unreliable repairman was proposed by Wang *et al.* [28]. They applied PSO algorithm to determine the optimal number of warm standbys and the joint optimal values for service rate and vacation rate, simultaneously to maximize the steady-state expected profit per unit time. Liou [21] examined an M/M/1 warm-standby MRP with multiple vacations and working breakdowns. The PSO algorithm was implemented to determine the optimal number of warm standbys and two variable service rates simultaneously at the optimal maximum profit. Yen *et al.* [30] studied a MRP with warm standbys and working breakdowns operating under the N policy. The steady-state probabilities of the number of failed machines in the system were derived using matrix-analytic method. They used two-stage optimization method to determine the optimal threshold N, and the joint optimal values for fast and slow service rates simultaneously at the optimal minimum cost. To the best of our knowledge, only a few researchers studied the MRP with warm standbys and working breakdowns under the N policy. Therefore, this motivates us to work on the retrial MRP with warm standbys and working breakdowns where the server operates under the N policy.

The purpose of this paper is threefold. Firstly, we employ matrix-analytic method to develop the steady-state probabilities which are used to calculate various system performance measures. Secondly, we establish a profit model to determine the optimum number of warm standbys S, and the joint optimal values for fast service and slow service rates, simultaneously by means of CPSO algorithm. Numerical examples are provided to illustrate CPSO. Thirdly, we carry out sensitivity analysis to examine the effects of system parameters on the optimal expected profit function.

2. Assumptions of the Model

We consider the N policy retrial MRP consisting of M operating machines with S warm standby machines in which the server is subject to working breakdowns. The definition of the N policy is to turn on the server when at least N ($N \geq 1$) failed machines are present in the system, and turn the server off only when none is present. The standby machine is referred to as warm standby when the failure rate is nonzero and is less than the failure rate of an operating machine. The assumptions of the model are described in the following.

(1) Operating machines are subject to failure according to independent Poisson process with parameter λ.
(2) Warm standby machines are subject to failure according to independent Poisson process with parameter θ ($0 < \theta < \lambda$).
(3) Once an operating machine fails, it is immediately replaced by an available spare. When a spare moves into an operating state, its failure characteristics will be that of an operating machine.

(4) When the server is turned on and working, the service times at this facility follow exponential distribution with parameter μ_1 for a fast service rate.
(5) When the server is turned on but subject to working breakdowns, the service times at this facility follow exponential distribution with parameter μ_2 for a slow service rate.
(6) Whenever a machine fails, it is immediately sent to the server and once a machine is repaired, it becomes as good as new.
(7) The server can serve only one failed machine at a time and the failed machines have to wait in the queue until the server is available.
(8) Breakdown times of the server are exponentially distributed with rate α.
(9) Repair times of the server are exponentially distributed with rate β.
(10) Retrial failed machines of the retrial queue repeats its request for service with an exponential amount of retrial time with rate γ.
(11) The failure times, the service times, the breakdown times, the repair times, and the retrial times are mutually independent of each other.
(12) Although slow service occurs during the working breakdown period, the failed machines continue arriving by a Poisson process. Once recovered to a normal situation, the server immediately serves failed machines with a fast service rate.

3. Steady-State Results

We represent the states of the system by the pair $(C(t), N(t))$, where $C(t)$ describes the status of the server and $N(t)$ is the number of failed machines in retrial orbit.

$C(t)=0$ denotes the server is in dormant state, where the server does not be activated until N failed machines are accumulated in retrial orbit.

$C(t)=1$ denotes the server is turned on and in idle state, where the failed machines in orbit can retrial.

$C(t)=2$ denotes the server is turned on and in busy state with service rate μ_1.

$C(t)=3$ denotes the server is turned on and in working breakdowns state with service rate μ_2.

$C(t)=4$ denotes the server is turned on and in working breakdowns state, where the failed machines in orbit can retrial.

The mean failure rate λ_n and the mean retrial rate γ_n are given by

$$\lambda_n = \begin{cases} M\lambda + (S-n)\theta, & 0 \le n < S \\ (M+S-n)\lambda, & S \le n < L = M+S \end{cases}$$

and

$$\gamma_n = n\gamma, \quad 1 \le n < L\,(= M + S).$$

For different states of the system, we define the steady-state probabilities as follows.

$P(0, n) \equiv$ probability of having n failed machines in retrial orbit when the server is in dormant state, where $n = 0, 1, 2, ..., \ N-1$;

$P(1, n) \equiv$ probability of having n failed machines in retrial orbit when the server is turned on and in idle state, and the failed machines in orbit can retrial, where $n = 1, 2, \ldots, \ L-1$;

$P(2, n) \equiv$ probability of having n failed machines in retrial orbit when the server is turned on and in busy state, where $n = 0, 1, 2, \ldots, \ L-1$;

$P(3, n) \equiv$ probability of having n failed machines in retrial orbit when the server is turned on and in working breakdowns state, where $n = 0, 1, 2, \ldots, \ L-1$;

$P(4, n) \equiv$ probability of having n failed machines in retrial orbit when the server is turned on and in working breakdowns state, and the failed machines in orbit can retrial, where $n = 1, 2, \ldots, \ L-1$.

3.1. Steady-state equations

The steady-state transition-rate diagram for the retrial MRP with working breakdowns operating under the N policy can be drawn as shown in Figure 1. The following steady-state equations are given by:

$$\lambda_0 P(0,0) = \mu_1 P(2,0) + \beta P(4,0), \tag{1}$$

$$\lambda_n P(0,n) = \lambda_{n-1} P(0,n-1), \quad 1 \le n \le N-1, \tag{2}$$

$$(\lambda_n + \gamma_n) P(1,n) = \mu_1 P(2,n) + \beta P(4,n), \quad 1 \le n \le N-1, \tag{3}$$

$$(\lambda_N + \gamma_N) P(1,N) = \lambda_{N-1} P(0,N-1) + \mu_1 P(2,N) + \beta P(4,N), \tag{4}$$

$$(\lambda_n + \gamma_n) P(1,n) = \mu_1 P(2,n) + \beta P(4,n), \quad N+1 \le n \le L-1, \tag{5}$$

$$(\lambda_1 + \mu_1 + \alpha) P(2,0) = \gamma_1 P(1,1) + \beta P(3,0), \tag{6}$$

$$(\lambda_{n+1} + \mu_1 + \alpha) P(2,n) = \lambda_n P(1,n) + \gamma_{n+1} P(1,n+1) + \lambda_n P(2,n-1) + \beta P(3,n), \quad 1 \le n \le L-2, \tag{7}$$

$$(\mu_1 + \alpha) P(2,L-1) = \lambda_{L-1} P(1,L-1) + \lambda_{L-1} P(2,L-2) + \beta P(3,L-1), \tag{8}$$

$$(\lambda_1 + \mu_2 + \beta) P(3,0) = \alpha P(2,0) + \lambda_0 P(4,0) + \gamma_1 P(4,1), \tag{9}$$

$$(\lambda_{n+1}+\mu_2+\beta)P(3,n)=\alpha P(2,n)+\lambda_n P(3,n-1)+\lambda_n P(4,n)+\gamma_{n+1}P(4,n+1),$$
$$1\le n\le L-2, \quad (10)$$

$$(\mu_2+\beta)P(3,L-1)=\alpha P(2,L-1)+\lambda_{L-1}P(3,L-2)+\lambda_{L-1}P(4,L-1), \quad (11)$$

$$(\lambda_0+\beta)P(4,0)=\mu_2 P(3,0), \quad (12)$$

$$(\lambda_n+\gamma_n+\beta)P(4,n)=\mu_2 P(3,n), \quad 1\le n\le L-1. \quad (13)$$

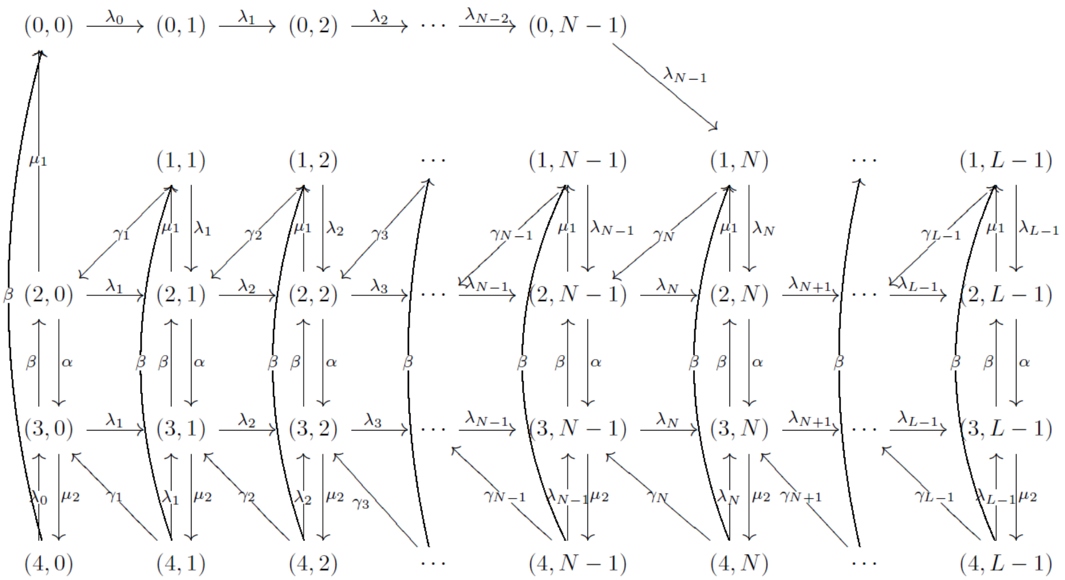

Figure 1. The state-transition-rate diagram for the retrial MRP with working breakdowns under the N policy.

3.2. Matrix-analytic method

Neuts [22] proposed a matrix-analytic method to analyze many complex queueing systems. We use this method to develop the steady-state probabilities $P(k,n)$ $(k=0,1,2,3,4)$. We establish the corresponding transition rate matrix Q of this Markov chain having the following block-tridiagonal form:

$$Q=\begin{bmatrix}\hat{B}_0 & \hat{C}_0 & & & & & & & & & & \\ \hat{A}_1 & B_1 & C_1 & & & & & & & & & \\ & A_2 & B_2 & C_2 & & & & & & & & \\ & & \ddots & \ddots & \ddots & & & & & & & \\ & & & A_{N-2} & B_{N-2} & C_{N-2} & & & & & & \\ & & & & A_{N-1} & B_{N-1} & \hat{C}_{N-1} & & & & & \\ & & & & & \hat{D}_N & E_N & F_N & & & & \\ & & & & & & D_{N+1} & E_{N+1} & F_{N+1} & & & \\ & & & & & & & \ddots & \ddots & \ddots & & \\ & & & & & & & & D_{L-2} & E_{L-2} & F_{L-2} \\ & & & & & & & & & D_{L-1} & E_{L-1}\end{bmatrix}$$

Each element of the matrix Q is listed as follows.

$$\hat{B}_0=\begin{bmatrix}-\lambda_0 & 0 & 0 & 0 \\ \mu_1 & -(\lambda_1+\mu_1+\alpha) & \alpha & 0 \\ 0 & \beta & -(\lambda_1+\mu_2+\beta) & \mu_2 \\ \beta & 0 & \lambda_0 & -(\lambda_0+\beta)\end{bmatrix},$$

$$\hat{C}_0=\begin{bmatrix}\lambda_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\hat{A}_1=\begin{bmatrix}0 & 0 & 0 & 0 \\ 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_1 & 0\end{bmatrix}, \qquad A_n=\begin{bmatrix}0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_n & 0\end{bmatrix}, \quad 2\le n\le N-1$$

$$B_n = \begin{bmatrix} -\lambda_n & 0 & 0 & 0 & 0 \\ 0 & -(\lambda_n + \gamma_n) & \lambda_n & 0 & 0 \\ 0 & \mu_1 & -(\lambda_{n+1} + \mu_1 + \alpha) & \alpha & 0 \\ 0 & 0 & \beta & -(\lambda_{n+1} + \mu_2 + \beta) & \mu_2 \\ 0 & \beta & 0 & \lambda_n & -(\lambda_n + \gamma_n + \beta) \end{bmatrix}, \ 1 \le n \le N-1$$

$$C_n = \begin{bmatrix} \lambda_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{n+1} & 0 & 0 \\ 0 & 0 & 0 & \lambda_{n+1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad 1 \le n \le N-2,$$

$$\hat{C}_{N-1} = \begin{bmatrix} \lambda_{N-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \lambda_N & 0 & 0 \\ 0 & 0 & \lambda_N & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \hat{D}_N = \begin{bmatrix} 0 & 0 & \gamma_N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_N & 0 \end{bmatrix},$$

$$D_n = \begin{bmatrix} 0 & \gamma_n & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_n & 0 \end{bmatrix}, \quad N+1 \le n \le L-1$$

$$E_n = \begin{bmatrix} -(\lambda_n + \gamma_n) & \lambda_n & 0 & 0 \\ \mu_1 & -(\lambda_{n+1} + \mu_1 + \alpha) & \alpha & 0 \\ 0 & \beta & -(\lambda_{n+1} + \mu_2 + \beta) & \mu_2 \\ \beta & 0 & \lambda_n & -(\lambda_n + \gamma_n + \beta) \end{bmatrix}, \ N \le n \le L-1,$$

$$F_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_{n+1} & 0 & 0 \\ 0 & 0 & \lambda_{n+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad N \le n \le L-2.$$

Let P denote the corresponding steady-state probability vector of Q. By partitioning the vector P as $P_n = [P_0, P_1, P_2, \ \ldots, P_{N-1}, P_N, \ \ldots, P_{L-1}]$, where

$P_0 = [P(0,0),\, P(2,0),\, P(3,0),\, P(4,0)]$,

$P_n = [P(0,n), P(1,n), P(2,n),\, P(3,n),\, P(4,n)]$, $1 \le n \le N-1$, and

$P_n = [P(1,n),\, P(2,n),\, P(3,n),\, P(4,n)]$, $N \le n \le L-1$,

are row vectors with dimensions 4, 5, 4, respectively. The steady-state equations $PQ = \mathbf{0}$ can be written as follows.

$$P_0 \hat{B}_0 + P_1 \hat{A}_1 = \mathbf{0}, \tag{14}$$

$$P_0 \hat{C}_0 + P_1 B_1 + P_2 A_2 = \mathbf{0}, \tag{15}$$

$$P_{n-1} C_{n-1} + P_n B_n + P_{n+1} A_{n+1} = \mathbf{0}, \ \ 2 \le n \le N-2 \tag{16}$$

$$P_{N-2} C_{N-2} + P_{N-1} B_{N-1} + P_N \hat{A}_N = \mathbf{0}, \ \ n = N-1 \tag{17}$$

$$P_{n-1} \hat{C}_{n-1} + P_n \hat{B}_n + P_{n+1} \hat{A}_{n+1} = \mathbf{0}, \quad N \le n \le L-2 \tag{18}$$

$$P_{L-2} \hat{C}_{L-2} + P_{L-1} \hat{B}_{L-1} = \mathbf{0}, \tag{19}$$

where $\mathbf{0}$ is a zero matrix.

3.3. Steady-state solutions

Taking some routine manipulations to (14)-(19), we finally get

$$P_0 = P_1(-\hat{A}_1 \hat{B}_0^{-1}) = P_1 X_0, \quad \text{where} \quad X_0 = -\hat{A}_1 \hat{B}_0^{-1}, \tag{20}$$

$$P_1 = P_2 X_1, \quad \text{where} \quad X_1 = -A_2 (X_0 \hat{C}_0 + B_1)^{-1}, \tag{21}$$

$$P_n = P_{n+1} X_n, \quad \text{where} \quad X_n = -A_{n+1}(X_{n-1} C_{n-1} + B_n)^{-1}, \ \ 2 \le n \le N-2 \tag{22}$$

$$P_{N-1} = P_N X_{N-1}, \ \text{where} \ \ X_{N-1} = -A_N (X_{N-2} C_{N-2} + B_{N-1})^{-1}, \tag{23}$$

$$P_n = P_{n+1} X_n, \ \text{where} \ \ X_n = -\hat{A}_{n+1}(X_{n-1} \hat{C}_{n-1} + \hat{B}_n)^{-1}, \ \ N \le n \le L-2 \tag{24}$$

$$P_{L-1}(X_{L-2} \hat{C}_{L-2} + \hat{B}_{L-1}) = \mathbf{0}. \tag{25}$$

Consequently, P_n $(0 \le n \le L-2)$ in (20)-(25) can be written as product form in terms of

P_{L-1}. The steady-state probability P_{L-1} can be found by using (25) and normalization equation, that is,

$$P_0 e + \sum_{n=1}^{N-1} P_n \hat{e} + \sum_{n=N}^{L-1} P_n e = 1,$$

where both $e = [1, 1, 1]^T$ and $\hat{e} = [1,1,1,1]^T$ are column vectors. Once the steady-state probability P_{L-1} is obtained, the steady-state probabilities $[P_0, P_1, P_2, P_3, \ldots, P_{L-1}]$ are solved from (20)-(24). The steps of the solution algorithm are described as follows.

3.4. The solution algorithm

Step 1. Set $X_0 = -\hat{A}_1 \hat{B}_0^{-1}$, $X_1 = -A_2(X_0 \hat{C}_0 + B_1)^{-1}$.

Step 2. For n from 2 to $N-2$, set $X_n = -A_{n+1}(X_{n-1}C_{n-1} + B_n)^{-1}$.

Step 3. For $n = N-1$, set $X_{N-1} = -\hat{A}_N (X_{N-2}C_{N-2} + B_{N-1})^{-1}$.

Step 4. For n from N to $L-2$, set $X_n = -\hat{A}_{n+1}(X_{n-1} \hat{C}_{n-1} + \hat{B}_n)^{-1}$.

Step 5. For k from 0 to $L-2$, set $\Phi_k = X_{L-2} X_{L-3} \cdots X_k$.

Step 6. Solve $P_{L-1}(X_{L-2} \hat{C}_{L-2} + \hat{B}_{L-1}) = \mathbf{0},$ $P_{L-1}\left[\Phi_0 e + \sum_{n=1}^{N-1} \Phi_n \hat{e} + \sum_{n=N}^{L-2} \Phi_n e + e\right] = 1$, and obtain steady-state probability P_{L-1}.

Step 7. Construct the steady-state probability P_n as follows:
if $0 \le n \le L-2$, assign $P_n = P_{L-1}\Phi_n$.

4. System Performance Measures

We define several system performance measures of the retrial MRP with working breakdowns operating under the N policy as follows:

$E[N_s] \equiv$ the expected number of failed machines in the system,
$E[S] \equiv$ the expected number of warm standby machines in the system,
$E[O] \equiv$ the expected number of operating machines in the system,
$\lambda_{eff} \equiv$ the effective failure rate.

After obtaining the steady-state solutions, we can calculate $E[N_s]$, $E[S]$, $E[O]$ and λ_{eff} from the following equations.

$$E[N_s] = \sum_{n=1}^{N-1} nP(0,n) + \sum_{n=1}^{L-1} n\left[P(1,n) + P(2,n) + P(3,n) + P(4,n)\right] \tag{26}$$

$$E[S]=\begin{cases}\sum_{n=0}^{S}(S-n)P(0,n), & N\geq S;\\ \sum_{n=0}^{N-1}(S-n)P(0,n)+\sum_{n=1}^{S}(S-n)P(1,n) & \\ +\sum_{n=0}^{S-1}(S-n-1)[P(2,n)+P(3,n)+P(4,n)], & N<S.\end{cases} \tag{27}$$

$$E[O]=L-E[N_s]-E[S], \tag{28}$$

$$\lambda_{eff}=\sum_{n=0}^{N-1}\lambda_n P(0,n)+\sum_{n=1}^{L-1}\lambda_n P(1,n)+\sum_{n=0}^{L-2}\lambda_{n+1}[P(2,n)+P(3,n)+P(4,n)]. \tag{29}$$

Thus the expected waiting time in the system, W_s, is given by

$$W_s=\frac{E[N_s]}{\lambda_{eff}}. \tag{30}$$

5. Profit Optimization Analysis

There are numerous researchers who have contributed to the study of MRP operating under different threshold control policies, namely, N policy, F policy, bi-level policy, recovery policy, and so on. In the MRP, most of the researchers have aimed at determining the optimal threshold policy or the optimal system parameters, such as the optimal number of spares, the optimum number of servers, the optimal service rate, the optimal balking rate, the optimal retrial rate, and so on. We construct the total expected profit function per unit time for the retrial MRP with working breakdowns operating under the N policy, where three decision variables S, μ_1, and μ_2 are considered. The discrete variable S should be a natural number but the continuous variables μ_1 and μ_2 are positive real. Our objective is to find the optimum value of (S, μ_1, μ_2), say (S^*, μ_1^*, μ_2^*), so as to maximize the profit function. Let us define

$p \equiv$ revenue per unit time when one machine is in an operating state,

$C_1 \equiv$ cost per unit time when one machine is in an operating state,

$C_2 \equiv$ cost per unit time when one machine is functioning as a warm standby,

$C_3 \equiv$ cost per unit time of providing the service rate μ_1,

$C_4 \equiv$ cost per unit time of providing the service rate μ_2,

$C_5 \equiv$ cost per unit time when one machine is waiting in the system.

Then the total expected profit function per unit time for the N policy retrial MRP as follows.

$$F(S,\mu_1,\mu_2)=(p-C_1)E[N_s]-C_2(S-E[S])-C_3\mu_1-C_4\mu_2-C_5W_s. \quad (31)$$

The profit maximization problem can be presented mathematically as

$$\underset{S,\,\mu_1,\,\mu_2}{Maximize}\, F(S,\mu_1,\mu_2).$$

5.1. Sensitivity analysis

We consider the revenue and cost elements for computational examples as follow: $p=\$100, C_1=\$50, C_2=\$25, C_3=\$20, C_4=\$10, C_5=\$15.$

To investigate the effect of various parameters on the profit function, we first give a graphical analysis in six cases for $M=15$ and $N=5$ with various values of $S=3, 6, 9,$ respectively.

Case 1: $\mu_1=3.0, \mu_2=1.5, \alpha=1.5, \beta=3.0, \theta=0.1, \gamma=0.5,$ λ varies from 0.15 to 0.5.

Case 2: $\lambda=0.15, \mu_2=1.5, \alpha=1.5, \beta=3.0, \theta=0.1, \gamma=0.5,$ μ_1 varies from 1.5 to 3.0.

Case 3: $\lambda=0.15, \mu_1=3.0, \mu_2=1.5, \beta=3.0, \theta=0.1, \gamma=0.5,$ α varies from 0.1 to 3.0.

Case 4: $\lambda=0.15, \mu_1=3.0, \mu_2=1.5, \alpha=1.5, \theta=0.1, \gamma=0.5,$ β varies from 0.1 to 3.0.

Case 5: $\lambda=0.15, \mu_1=3.0, \mu_2=1.5, \alpha=1.5, \beta=3.0, \gamma=0.5,$ θ varies from 0.01 to 0.15.

Case 6: $\lambda=0.15, \mu_1=3.0, \mu_2=1.5, \alpha=1.5, \beta=3.0, \theta=0.1,$ γ varies from 0.1 to 1.0.

Figures 2-7 depict the sensitivity performance of profit function F on $\lambda, \mu_1, \alpha, \beta, \theta,$ and γ, respectively. We observe from Figure 2 that (i) $\partial F/\partial\lambda$ is negative which means that F is decreasing on λ for all S; (ii) $\partial F/\partial\lambda$ has the same tendency for all S; and (iii) as λ is fixed, $\partial F/\partial\lambda$ is getting smaller as S increases. It reveals from Figure 3 that (i) $\partial F/\partial\mu_1$ is positive which means that F is increasing on μ_1 for all S; (ii) $\partial F/\partial\mu_1$ has the same tendency for all S; and (iii) as μ_1 is fixed, $\partial F/\partial\mu_1$ is getting larger as S increases. From Figure 4, it is clear that (i) $\partial F/\partial\alpha$ is negative which means that F is decreasing on α for all S; (ii) $\partial F/\partial\alpha$ has the same tendency for all S; and (iii) as α is fixed, $\partial F/\partial\alpha$ is getting smaller as S increases. Figure 5 shows that (i) $\partial F/\partial\beta$ is positive which means that F is increasing on β or all S; (ii) $\partial F/\partial\beta$ has the same tendency for all S; and (iii) as β is fixed, $\partial F/\partial\beta$ is getting larger as S increases. It appears from Figure 6 that (i) $\partial F/\partial\theta$ is negative which means that F is decreasing on θ for all S; and (ii) as θ is fixed, $\partial F/\partial\theta$ is getting smaller as S increases. It is seen in Figure 7 that (i) $\partial F/\partial\gamma$ is positive which means that F is increasing on γ for all S; (ii) $\partial F/\partial\gamma$ has the same tendency for all S; and (iii) as γ is fixed, $\partial F/\partial\gamma$ is getting larger as S increases.

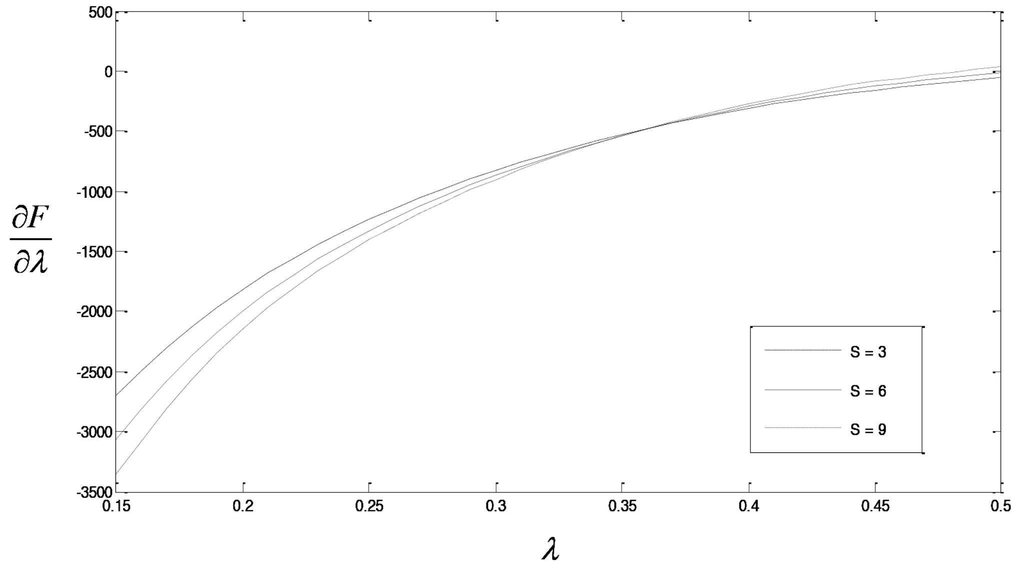

Figure 2. Sensitivity analysis of F with respect to λ for different S, where $\mu_1 = 3.0, \mu_2 = 1.5, \alpha = 1.5, \beta = 3.0, \theta = 0.1, \gamma = 0.5$.

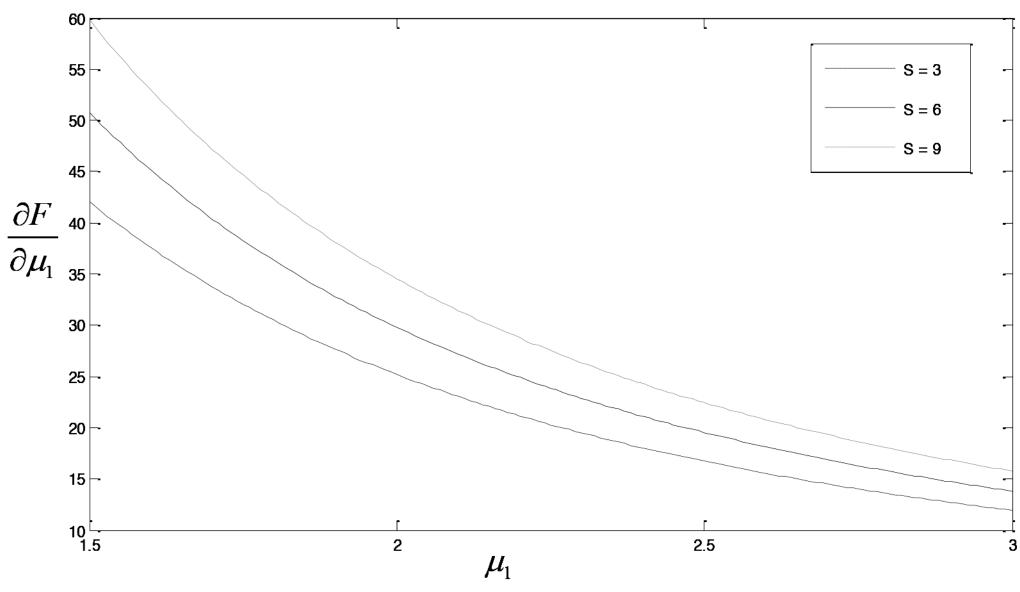

Figure 3. Sensitivity analysis of F with respect to μ_1 for different S, where $\lambda = 0.15, \mu_2 = 1.5, \alpha = 1.5, \beta = 3.0, \theta = 0.1, \gamma = 0.5$.

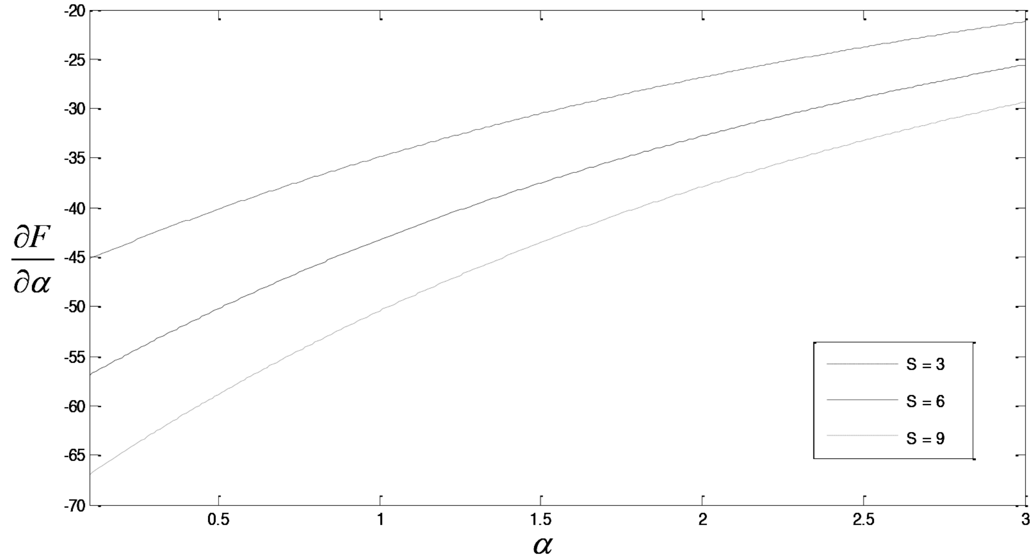

Figure 4. Sensitivity analysis of F with respect to α for different S, where $\lambda = 0.15$, $\mu_1 = 3.0$, $\mu_2 = 1.5$, $\beta = 3.0$, $\theta = 0.1$, $\gamma = 0.5$.

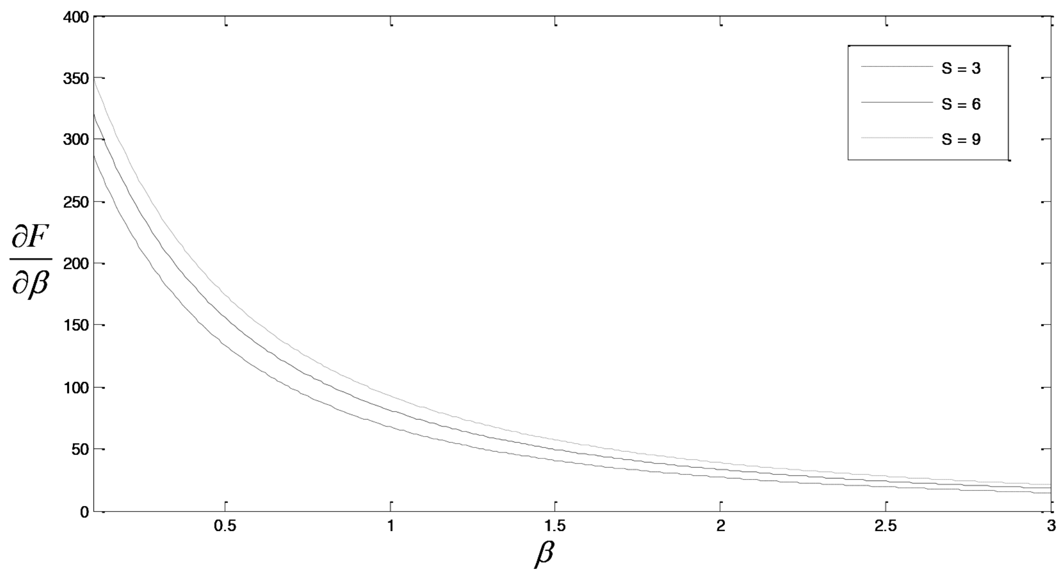

Figure 5. Sensitivity analysis of F with respect to β for different S, where $\lambda = 0.15$, $\mu_1 = 3.0$, $\mu_2 = 1.5$, $\alpha = 1.5$, $\theta = 0.1$, $\gamma = 0.5$.

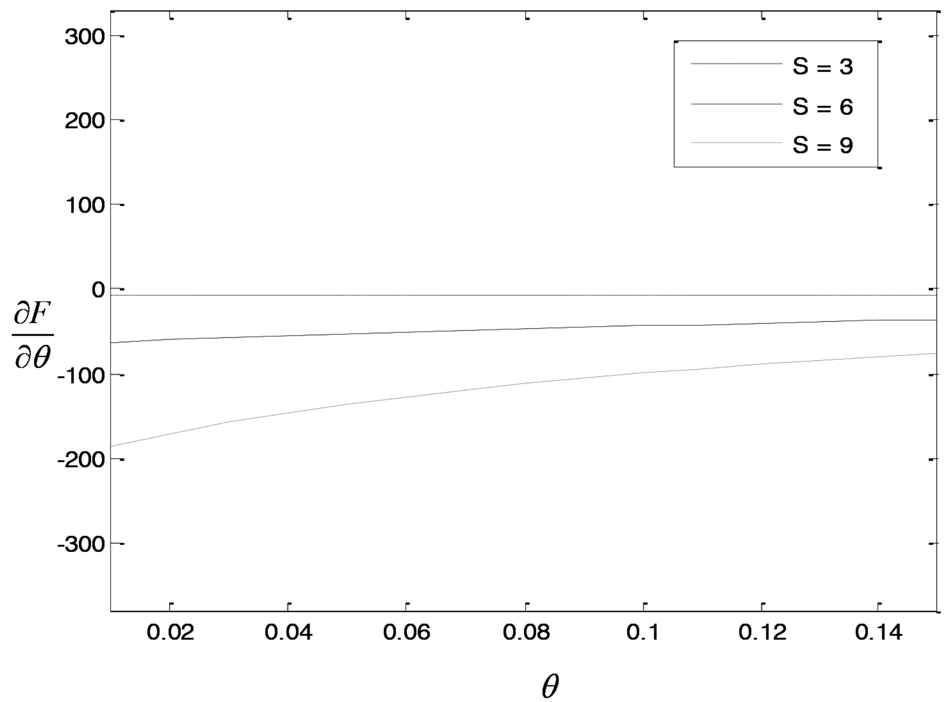

Figure 6. Sensitivity analysis of F with respect to θ for different S, where $\lambda = 0.15, \mu_1 = 3.0, \mu_2 = 1.5, \alpha = 1.5, \beta = 3.0, \gamma = 0.5.$

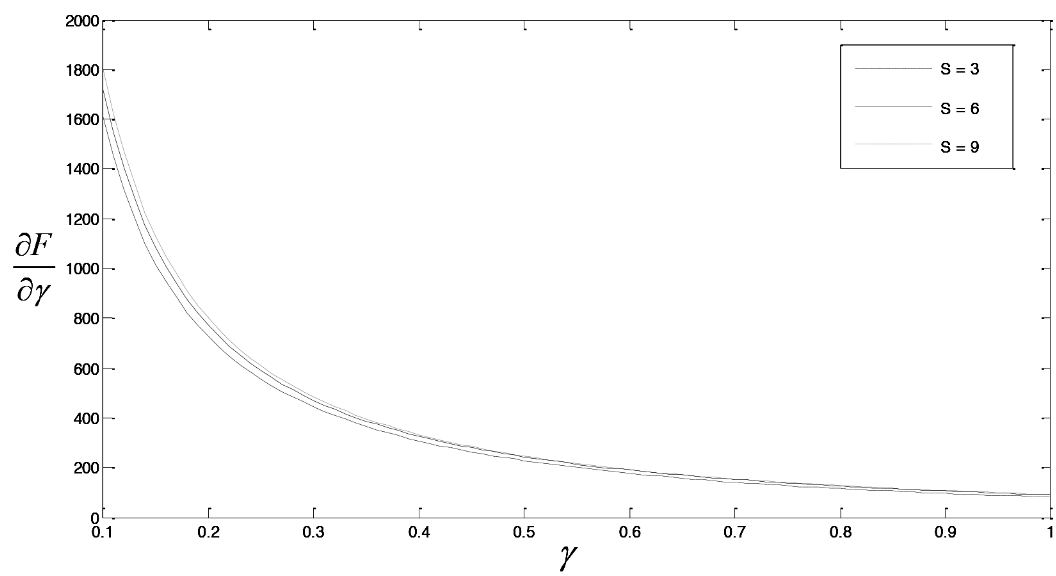

Figure 7. Sensitivity analysis of F with respect to γ for different S, where $\lambda = 0.15, \mu_1 = 3.0, \mu_2 = 1.5, \alpha = 1.5, \beta = 3.0, \theta = 0.1.$

5.2. Profit optimization

The particle Swarm Optimization (PSO) algorithm was firstly presented by Kennedy and Eberhart [17] in 1995. Under some necessary conditions, the global convergence analysis is summarized in Solis and Wets [24] and Bergh and Engelbrecht [4]. In this paper we use the Canonical particle swarm optimization (CPSO) algorithm (see Carlisle and Dozier [5]) which has improved performance and preferred properties in convergence.

When the basic PSO algorithm is used to solve an optimization problem with D decision variables, the solution space can be viewed as a D dimensional space. Generally, $X_i = (x_{i1}, x_{i2}, \cdots, x_{id}, \cdots, x_{iD})$ is denoted as the position of particle i and the value of dth variable x_{id} is the position of particle i in the dth dimension. Meanwhile particle i moves forward according to its own direction and step-size, i.e., velocity $V_i = (v_{i1}, v_{i2}, \cdots, v_{id}, \cdots, v_{iD})$, which is randomly generated by its best position $pbest$ and globally best position $gbest$.

The formulae that update the position x_{id} and velocity v_{id} are as follow:

$$v_{id} = \begin{cases} K(v_{id} + c_1 \times rand() \times (p_{id} - x_{id}) + c_2 \times rand() \times (p_{gd} - x_{id})), & X_{\min} < x_{id} < X_{\max}, \\ 0, & otherwise, \end{cases} \tag{32}$$

and

$$x_{id} = \begin{cases} x_{id} + v_{id}, & X_{\min} \le x_{id} + v_{id} \le X_{\max}, \\ X_{\max}, & x_{id} + v_{id} > X_{\max}, \\ X_{\min}, & x_{id} + v_{id} < X_{\min}, \end{cases} \tag{33}$$

where (i) the constriction factor $K = \dfrac{2}{|2 - C - \sqrt{C^2 - 4C}\,|}$, $C = c_1 + c_2, c_1 = 2.8, c_2 = 1.3$, and (ii) *rand*() is random variable in [0,1], p_{id} is the best position of particle i in dimension d and p_{gd} is the globally best position of flock in dimension d, $X_{\min}$ and $X_{\max}$ which are the lower bound and the upper bound of the component of vector X_i, respectively. The process for implementing the CPSO algorithm is as follows:

Step 1. Set initial parameters including the population size $Pnum$, the dimension of solution space D, the number of iteration M, $X_{\min}$, $X_{\max}$.

Step 2. Set randomly initial positions X_0 and velocities V_0 of all particles.

Step 3. Evaluate the benefit function values of all particles and define the initial particle best positions $pbest$ and global best position $gbest$.

Step 4. Update the velocities and positions of all particles according to the equations (32)-(33).

Step 5. Update the benefit function values of all articles and the corresponding *pbest* and *gbest* .

Step 6. If the termination criterion is reached, usually a maximum number of iterations, stop and return the consequence list; otherwise, go back to Step 4.

In this application, the number of particles is $Pnum = 100$, the number of iterations is $M = 2000$, the lower bound $X_{\min}$ and upper bound $X_{\max}$ of decision variables are 0 and 3, respectively. All examples in our paper are repeated 10 times and each run has the almost identical result which illustrates the CPSO algorithm is robust to our model.

We set $M = 15,\ 1 \le S \le 10,\ 0.1 \le \mu_1 \le 10,\ 0.1 \le \mu_2 \le 10,\ \mu_2 < \mu_1.$ As the following numerical examples, we consider $p = \$100,\ C_1 = \$50,\ C_2 = \$25,\ C_3 = \$20,\ C_4 = \$10,$ and $C_5 = \$15.$ The detailed optimal solutions $F(S^*, \mu_1^*, \mu_2^*)$ (= Max{Profit}) and related parameters are shown in Tables 1-6. One sees rom Tables 1-6 that (i) the number of warm standbys S^* increases as N or λ or γ or θ increases; (ii) the number of warm standbys S^* are the same even though α varies from 1.0 to 2.5 or β varies from 2.0 to 4.0; (iii) $F(S^*, \mu_1^*, \mu_2^*)$ decreases as N or λ or α or θ increases; and (iv) $F(S^*, \mu_1^*, \mu_2^*)$ increases as β or γ increases. Intuitively, the optimum number of warm standbys S^* seems too insensitive to changes in α or β.

Table 1. The search results of numerical experiments by CPSO for various values of N.

$(N, \gamma, \theta, \alpha, \beta, \lambda)$	S^*	μ_1^*	μ_2^*	Max{Profit}
(3,0.3,0.1,1.5,3.0,0.15)	3	7.381	2.879	423.324
(5,0.3,0.1,1.5,3.0,0.15)	4	7.331	2.979	419.739
(10,0.3,0.1,1.5,3.0,0.15)	5	6.932	3.077	409.017

Table 2. The search results of numerical experiments by CPSO for various values of λ.

$(N, \gamma, \theta, \alpha, \beta, \lambda)$	S^*	μ_1^*	μ_2^*	Max{Profit}
(5,0.3,0.1,1.5,3.0,0.1)	4	5.067	2.351	498.875
(5,0.3,0.1,1.5,3.0,0.15)	4	7.331	2.979	419.739
(5,0.3,0.1,1.5,3.0,0.2)	2	8.478	3.131	341.092

Table 3. The search results of numerical experiments by CPSO for various values of θ.

$(N, \gamma, \theta, \alpha, \beta, \lambda)$	S^*	μ_1^*	μ_2^*	Max{Profit}
(5,0.3,0.05,1.5,3.0,0.15)	5	7.451	3.071	422.389
(5,0.3,0.1,1.5,3.0,0.15)	4	7.331	2.979	419.739
(5,0.3,0.15,1.5,3.0,0.15)	3	7.125	2.869	418.047

Table 4. The search results of numerical experiments by CPSO for various values of α.

$(N, \gamma, \theta, \alpha, \beta, \lambda)$	S^*	μ_1^*	μ_2^*	Max{Profit}
(5,0.3,0.1,1.0,3.0,0.15)	4	7.693	1.394	430.333
(5,0.3,0.1,1.5,3.0,0.15)	4	7.331	2.979	419.739
(5,0.3,0.1,2.5,3.0,0.15)	4	6.319	5.442	412.038

Table 5. The search results of numerical experiments by CPSO for various values of β.

$(N, \gamma, \theta, \alpha, \beta, \lambda)$	S^*	μ_1^*	μ_2^*	Max{Profit}
(5,0.3,0.1,1.5,2.0,0.15)	4	6.713	4.651	413.723
(5,0.3,0.1,1.5,3.0,0.15)	4	7.331	2.979	419.739
(5,0.3,0.1,1.5,4.0,0.15)	4	7.694	1.689	426.535

Table 6. The search results of numerical experiments by CPSO for various values of γ.

$(N, \gamma, \theta, \alpha, \beta, \lambda)$	S^*	μ_1^*	μ_2^*	Max{Profit}
(5,0.1,0.1,1.5,3.0,0.15)	2	9.154	2.988	277.693
(5,0.3,0.1,1.5,3.0,0.15)	4	7.331	2.979	419.739
(5,0.5,0.1,1.5,3.0,0.15)	4	6.251	2.974	464.383

6. Conclusions

This paper analyzes a retrial machine repair problem with working breakdowns under N policy. The server is subject to working breakdowns when there is at least one failed machine in the system. To derive the steady-state probabilities of the system as well as several system performance measures, a matrix-analytic method is employed. We carry out the sensitivity analysis of these measures with respect to various system parameters. Moreover, we construct a profit model to determine the optimum number of warm standbys S, and the joint optimal values for fast service and slow service rates, simultaneously by using Canonical particle swarm optimization algorithm.

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